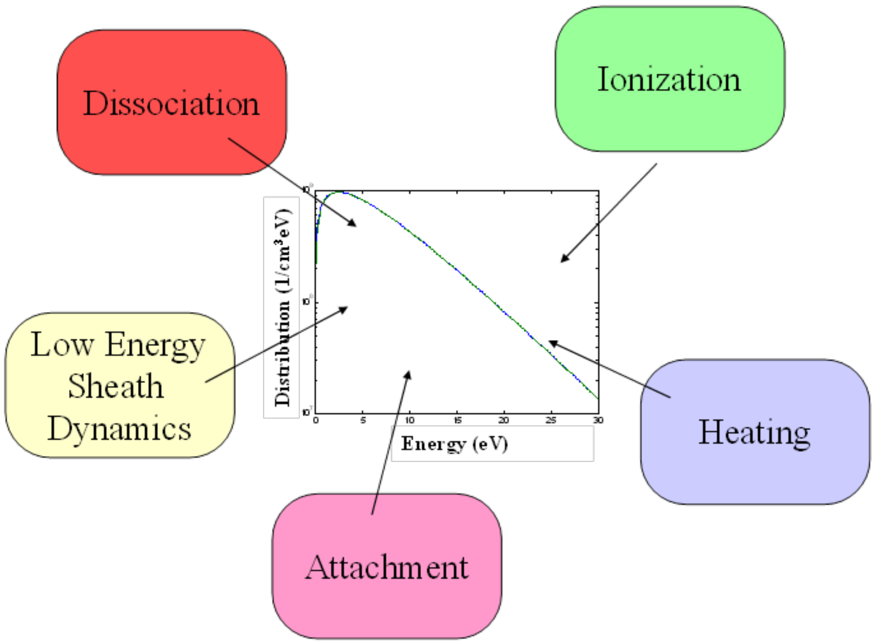
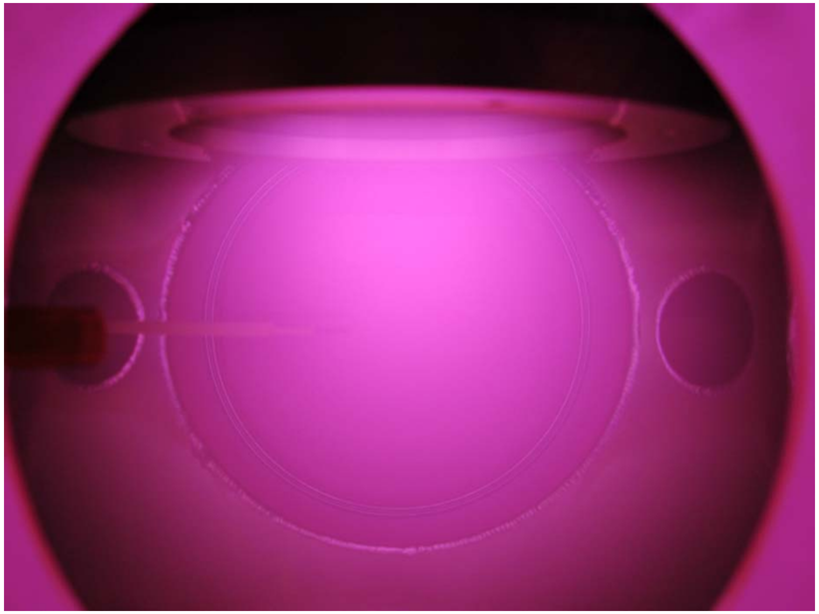
Geometry considerations for EEDF measurements using Langmuir probes

Steven Shannon
Associate Professor
NCSU Nuclear Engineering

NC-AVS MPM Meeting
10 November 2011
San Jose, California

Overview

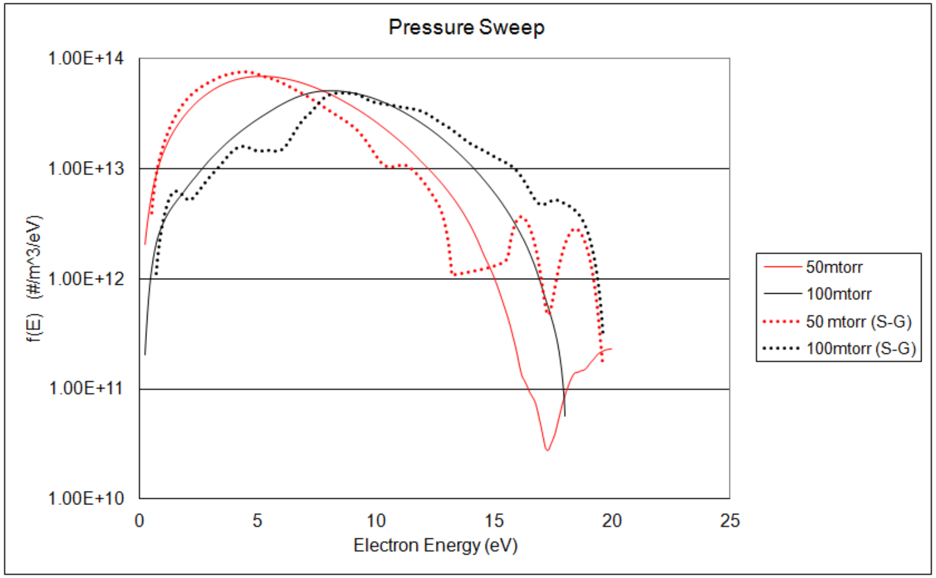




- EEDF's influence the most critical aspects of low temperature plasmas
- Accurate measurement of EEDF's over a large energy range are difficult, and the only practical way to measure them is still with Langmuir probes
- This talk focuses on some analysis challenges due to the geometry dependence of EEDF measurement using finite dimension single tipped Langmuir probes
- We will introduce a geometry dependent formulation for electron current for a biased cylindrical probe and spherical probe and discuss the ramifications of this geometry dependence on accurate reconstruction of EEDF's from probe IV characteristics

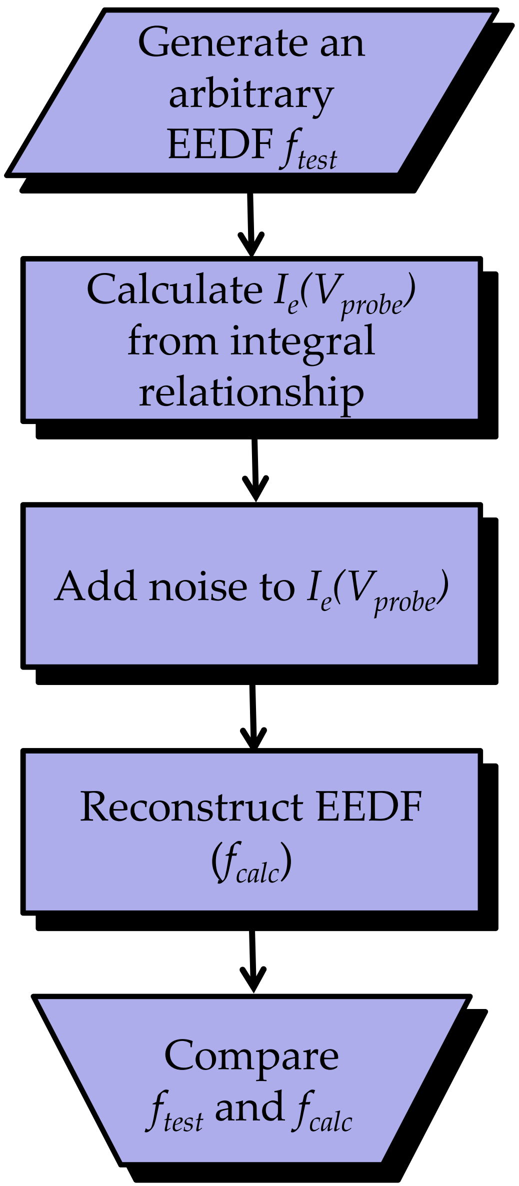
What we have been up to at NCSU – demonstrating high accuracy analysis techniques to obtain EEDF's

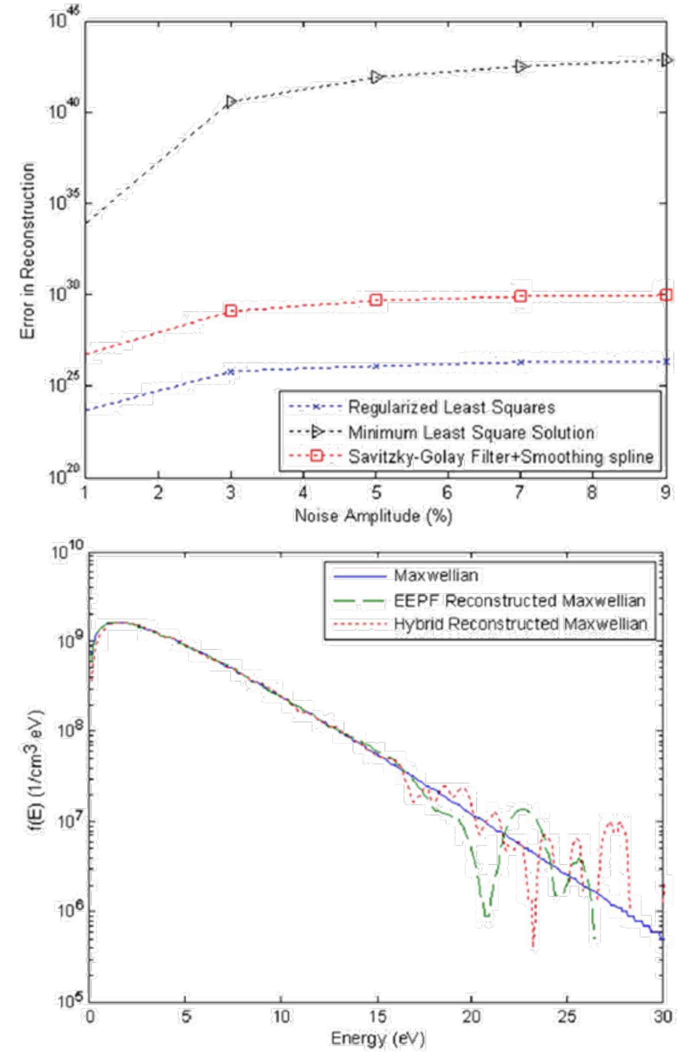
- Since the relationship between the EEDF and the measured probe data is an integral problem, noise amplification of the solution can be problematic
- Data smoothing techniques, rigorous point by point differentiation, and analog conditioning are the most commonly used techniques to address this
- Using integral techniques, more accurate distributions are obtained



$$f(E)_{E=-eV_{probe}} = -\frac{4}{A_p e^2}\sqrt{-\frac{m_e V_{probe}}{2e}}\frac{d^2 I_e}{dV_{probe}^2}$$

Numerical exercise

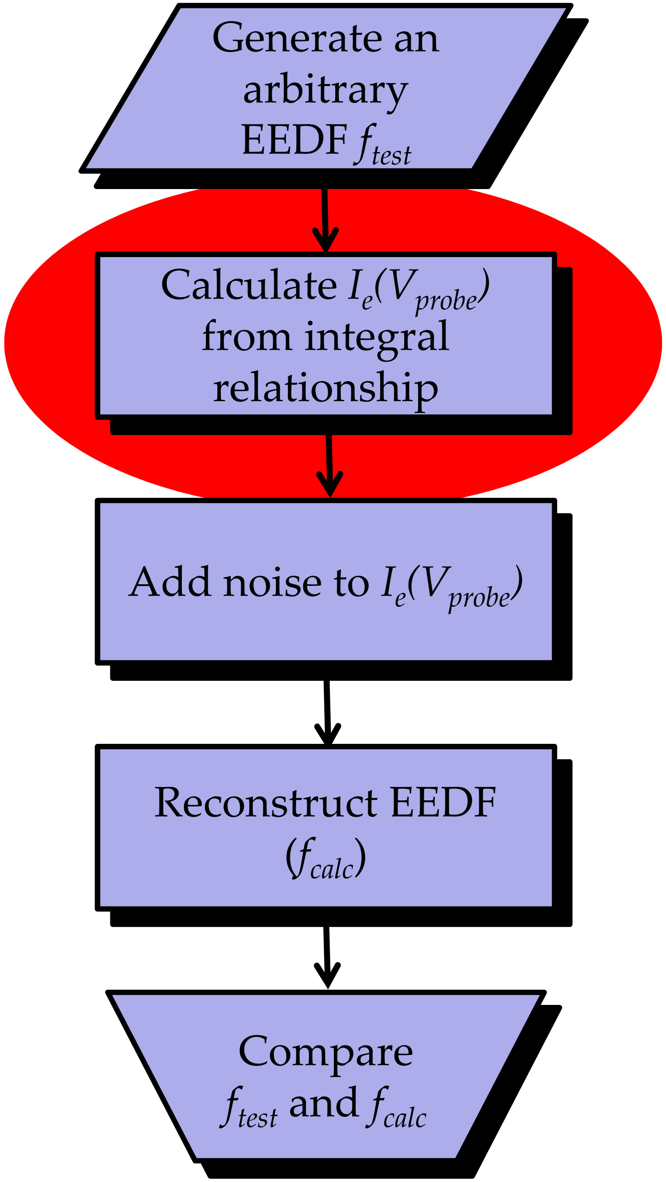


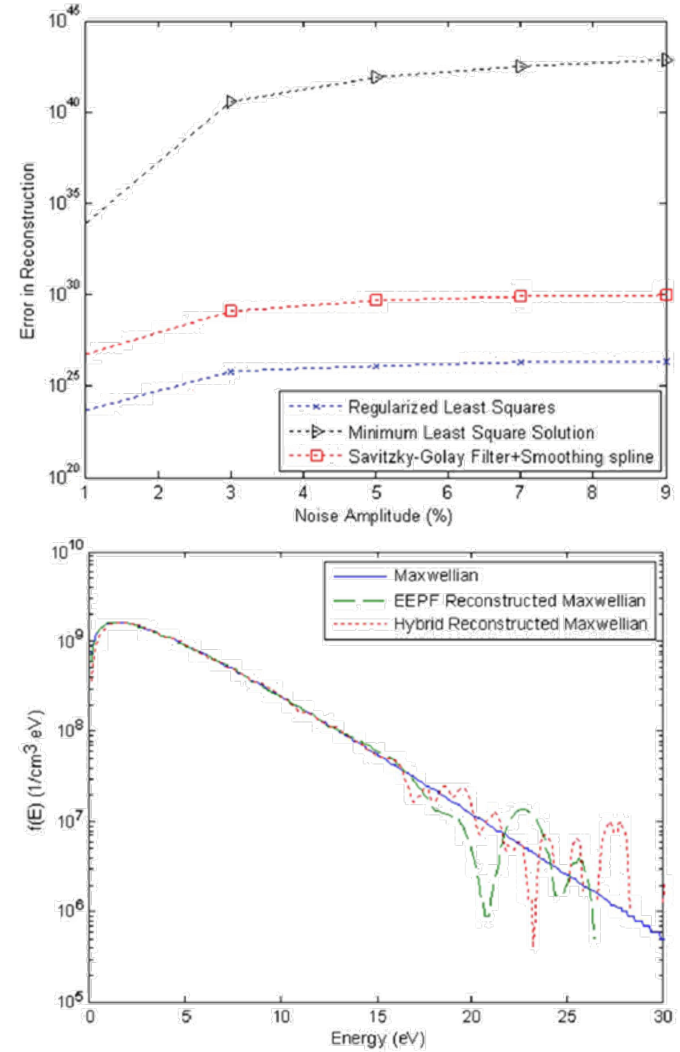


El Saghir, Kennedy, Shannon (2010). *IEEE Trans. Plasma Sci.* ***38****(2)* pp. 156-162

El Saghir, Shannon (2011), *IEEE Trans. Plasma Sci.* ***39****(1)* pp. 596-602

Numerical exercise

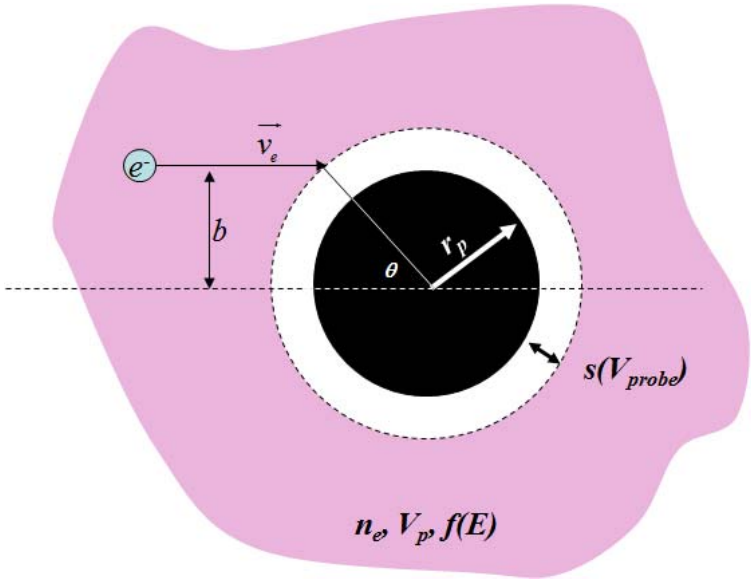




El Saghir, Kennedy, Shannon (2010). *IEEE Trans. Plasma Sci.* ***38****(2)* pp. 156-162

El Saghir, Shannon (2011), *IEEE Trans. Plasma Sci.* ***39****(1)* pp. 596-602

The relationship between electron current and EEDF



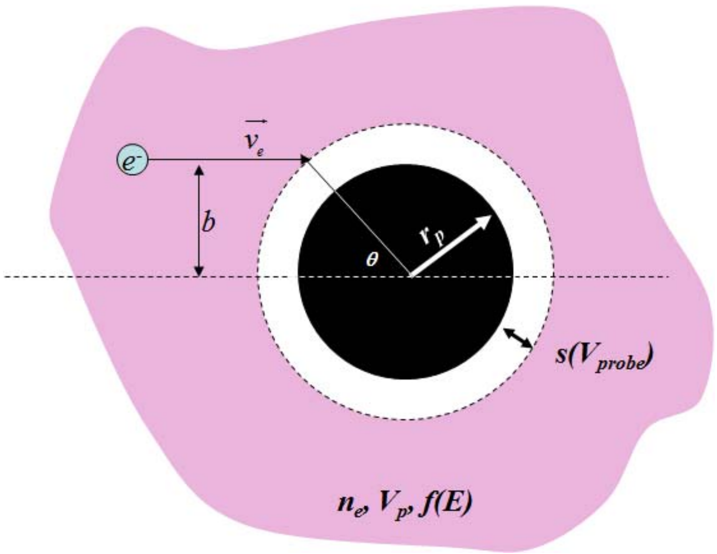
- As an electron approaches a surface with a retarding potential V_p, it will either be collected or repelled based on it's velocity vector relative to the retarding potential.
- If the perpendicular component of the velocity provides sufficient energy to overcome the potential, the electron is collected.
- By assuming isotropic distributions, one can integrate over all possible incident angles, accounting for the "perpendicular energy component" relative to the potential, and obtain an equation for electron current.

Depending on your probe geometry, this collection integral is different

Spherical Probes

$$I_e(V_{probe}) = -\frac{A_p e}{2}\sqrt{\frac{2}{m}}\int_{E=V_{probe}}^{\infty} f(E)\sqrt{E}\int_{b=0}^{r_p\sqrt{1-\frac{V_{probe}}{E}}} b\,db\,dE$$

$$I_e\left(V_{probe}\right) = \frac{-A_p e}{4}\sqrt{\frac{2}{m_e}}\int_{E=V_{probe}}^{\infty} f(E)\sqrt{E}\left(1-\frac{eV_{probe}}{E}\right)dE$$



Cylindrical Probes

$$I_e\left(V_{probe}\right) = \frac{-A_p e}{2}\int_{E=V_{probe}}^{\infty} f(E)\sqrt{\frac{2E}{m}}\int_{b=0}^{r_p\sqrt{1-\frac{V_{probe}}{E}}} db\,dE$$

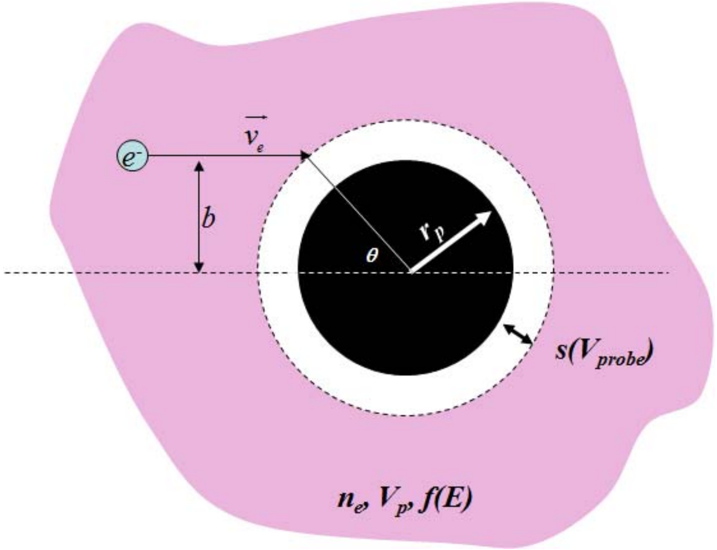
$$I_e\left(V_{probe}\right) = \frac{-A_p e}{2}\sqrt{\frac{2}{m}}\int_{E=V_{probe}}^{\infty} f(E)\sqrt{E}\left(1-\frac{eV_{probe}}{E}\right)^{1/2} dE$$

Depending on your probe geometry, this collection integral is different

Spherical Probes

$$I_e\left(V_{probe}\right) = -\frac{A_p e}{2}\sqrt{\frac{2}{m}}\int_{E=V_{probe}}^{\infty} f(E)\sqrt{E}\int_{b=0}^{r_p\sqrt{1-\frac{V_{probe}}{E}}} b\,db\,dE$$

$$I_e\left(V_{probe}\right) = \frac{-A_p e}{4}\sqrt{\frac{2}{m_e}}\int_{E=V_{probe}}^{\infty} f(E)\sqrt{E}\left(1-\frac{eV_{probe}}{E}\right)dE$$



Cylindrical Probes

$$I_e\left(V_{probe}\right) = \frac{-A_p e}{2}\int_{E=V_{probe}}^{\infty} f(E)\sqrt{\frac{2E}{m}}\int_{b=0}^{r_p\sqrt{1-\frac{V_{probe}}{E}}} db\,dE$$

$$I_e\left(V_{probe}\right) = \frac{-A_p e}{2}\sqrt{\frac{2}{m}}\int_{E=V_{probe}}^{\infty} f(E)\sqrt{E}\left(1-\frac{eV_{probe}}{E}\right)^{1/2} dE$$

Mott-Smith and Langmuir (1926). *Physical Review* **28**(4), 727-763

In 1930, the more commonly used "Druyvesteyn Relation" was derived for spherical probes (Physica **10** 61)

$$I_e = -\frac{A_p e}{4}\sqrt{\frac{2}{m_e}}\int_{-eV_{probe}}^{\infty}\frac{f(E)\left(E+eV_{probe}\right)}{\sqrt{E}}dE \quad \text{or} \quad f(E)_{E=-eV_{probe}} = -\frac{4}{A_p e^2}\sqrt{-\frac{m_e V_{probe}}{2e}}\frac{d^2 I_e}{dV_{probe}^2}$$

- This was only derived for the spherical geometry
- The cylindrical geometry does not have a nice analytical differential form like the spherical solutions does

- In his paper, Druyvesteyn makes the assumption of geometry invariance. This assumption was further developed by Kagan and Purel in the 1960's for Maxwell-Boltzmann distribution functions
- Since then, the second derivative form has been the way to get EEDF's

However, several works over the years have hinted that this may not carry over to any geometry

- Mott-Smith and Lanmguir reported a "geometry dependent effect" on electron current where current density dropped by a factor of $(1+2e\Phi/mv^2)$ for a spherical probe and $(1+2e\Phi/mv^2)^{1/2}$ for the cylindrical case (Phys. Rev. **28**, 727 1926)
- Emeleus reported that electron deflection may have relevance when the distribution is "sharply peaked" (Physical Letters **71A**, Nos. 2,3 1979)
- Hoskinson and Hershkowitz reported "an electron current that increases more quickly with voltage than in the ideal orbital motion limited theory" that they attributed to finite probe length effects, but none-the-less suggested a dependence between electron current collection and probe geometry (Plasma Sources Sci. Technol. **15** pp. 85–90)
- Knappmiller and Robertson presented a differential cylindrical formulation for EVDF that matched planar probe data analyzed with the Druyvesteyn relation (Phys. Rev. E **73**, 066402 (2006)

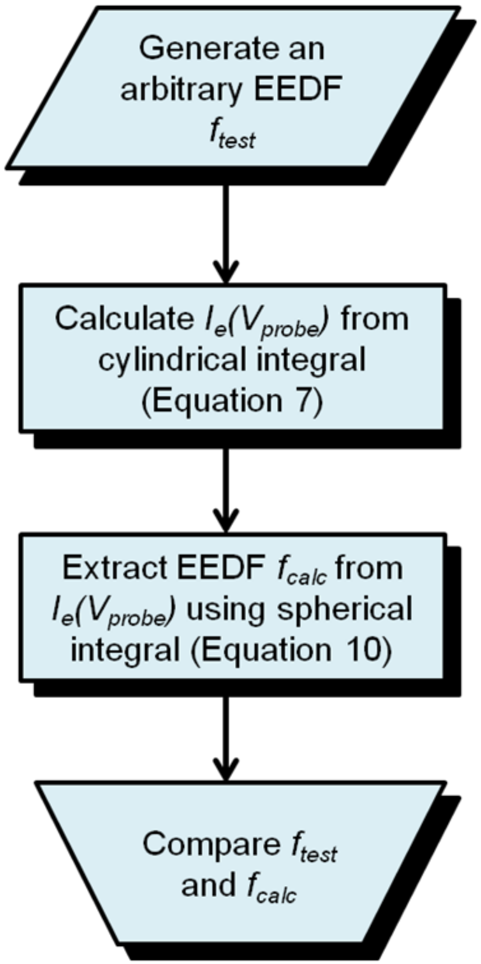
Is the Druyvesteyn relation really geometry invariant… and if it is not, how does this impact EEDF analysis?

Is this a big deal or can we ignore it?

$$I_e = -\frac{A_p e}{4}\sqrt{\frac{2}{m_e}}\int_{-eV_{probe}}^{\infty}\frac{f(E)\left(E+eV_{probe}\right)}{\sqrt{E}}\,dE$$

Vs.

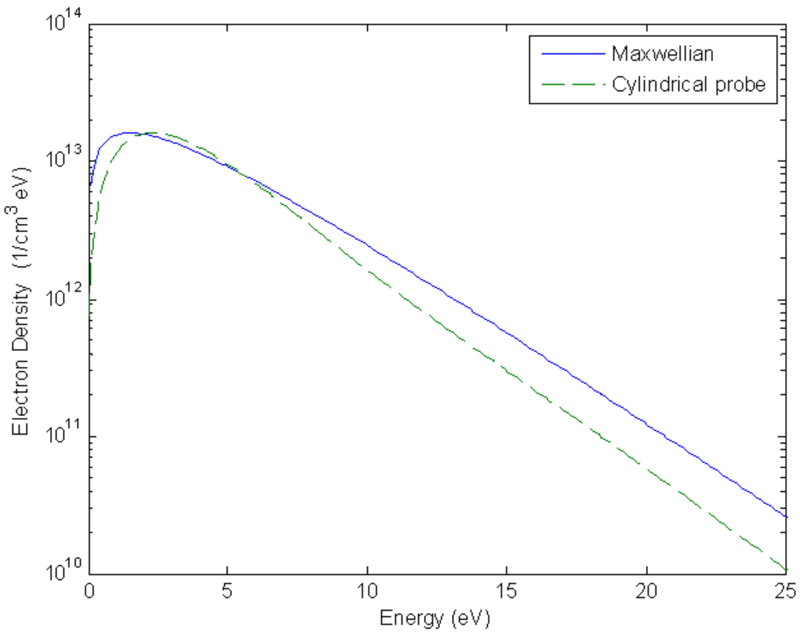
$$I_e\left(V_p\right) = \frac{-A_p e}{4\pi}2\pi\int_{E=V_p}^{\infty} f(E)\sqrt{\frac{2E}{m}}\int_{b=0}^{r_p\sqrt{1-\frac{V_p}{E}}} b\,db\,dE$$



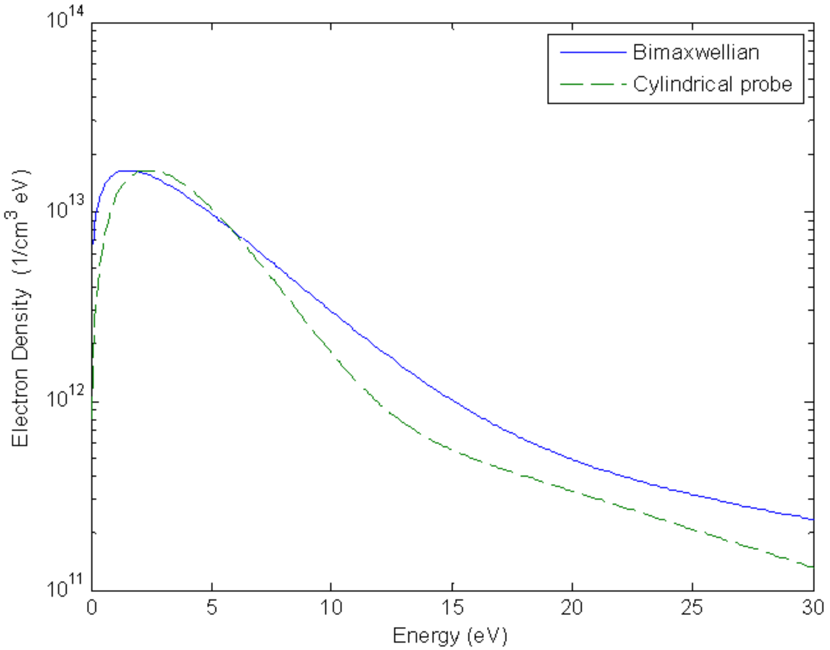
- Perform a similar numerical exercise that we did for reconstruction strategies
- The goal here is to mimic an experiment where someone is using a cylindrical probe with Druyvesteyn analysis and see how it impacts the EEDF measurement

Comparison of Maxwellian type distributions…

3 eV Maxwellian

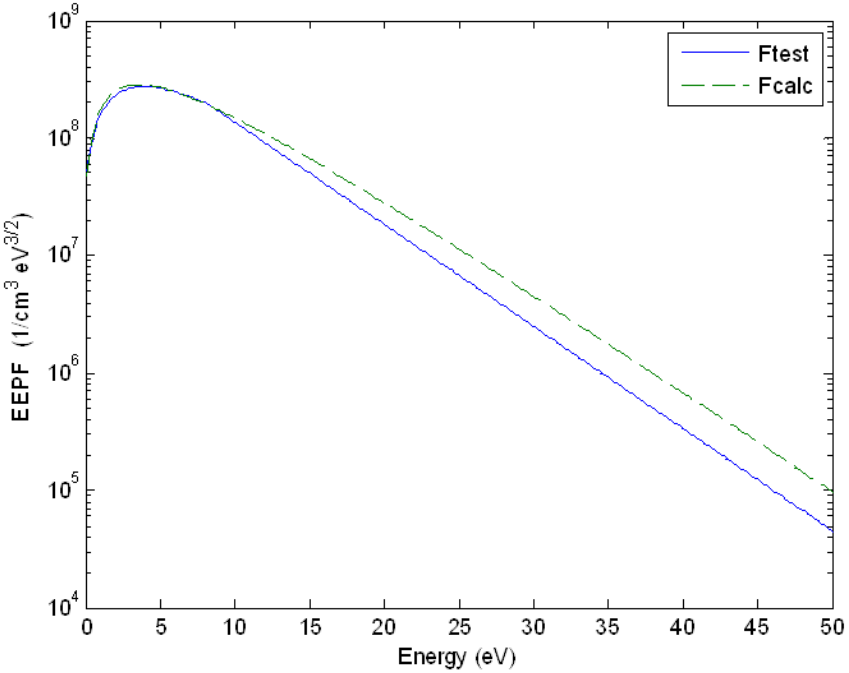


3 eV / 5eV Bi-Maxwellian

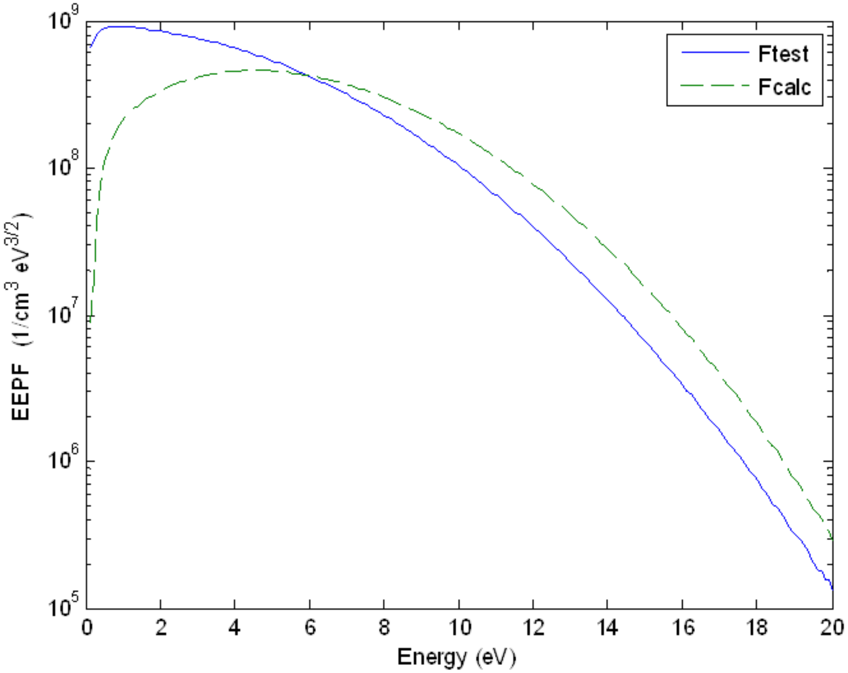


For most probe studies, this is probably not a killer. However, if we really do need to worry about energy distribution control, and probes are our only clue to what EEDF's look like at higher energies for model validation, etc… this could be important.

What about "non-Maxwellian" distributions?



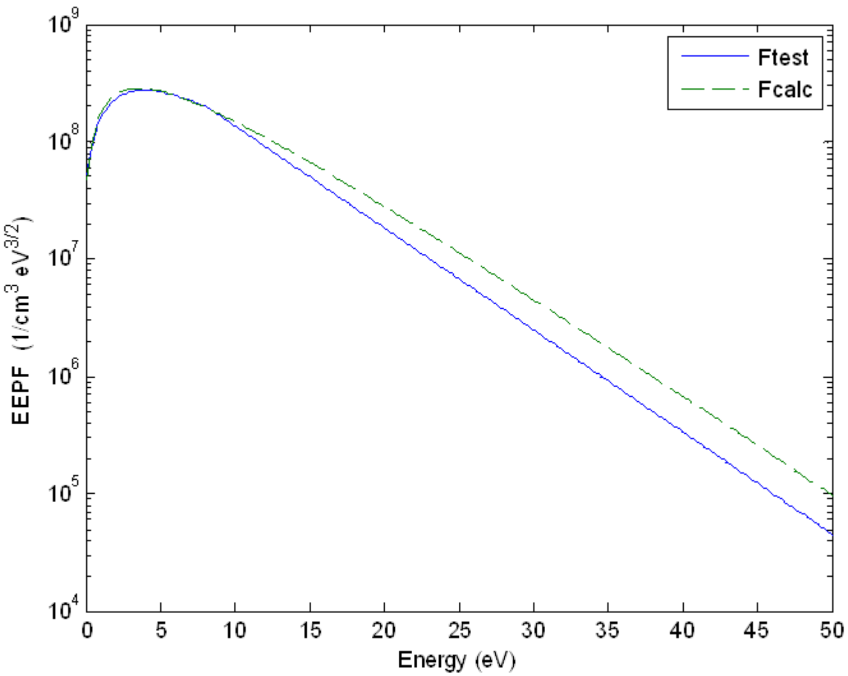
Maxwellian



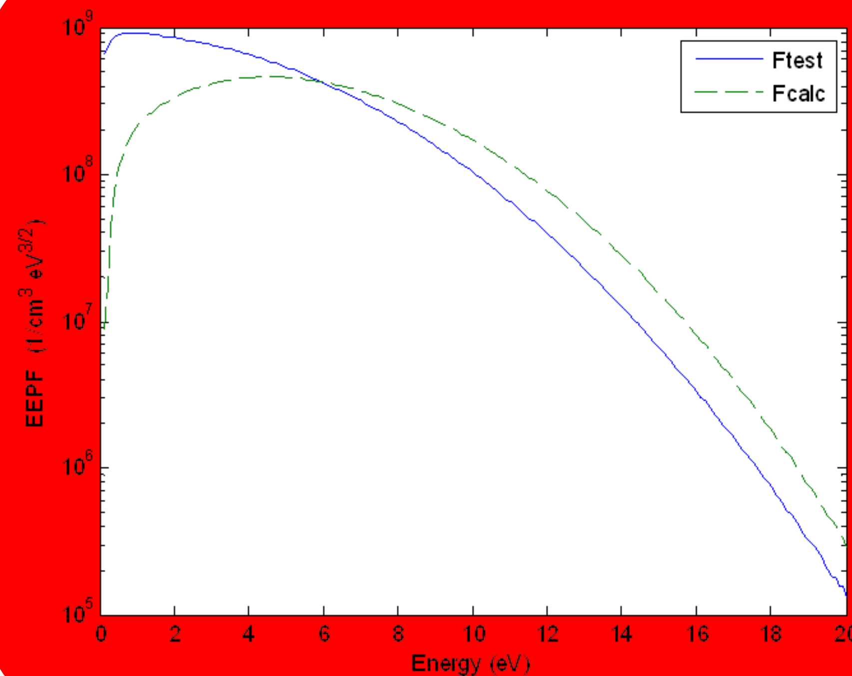
Druyvesteyn

Distortion appears to get worse as distribution becomes less "Maxwellian"… supporting Emeleus work from the 1950's

What about "non-Maxwellian" distributions?

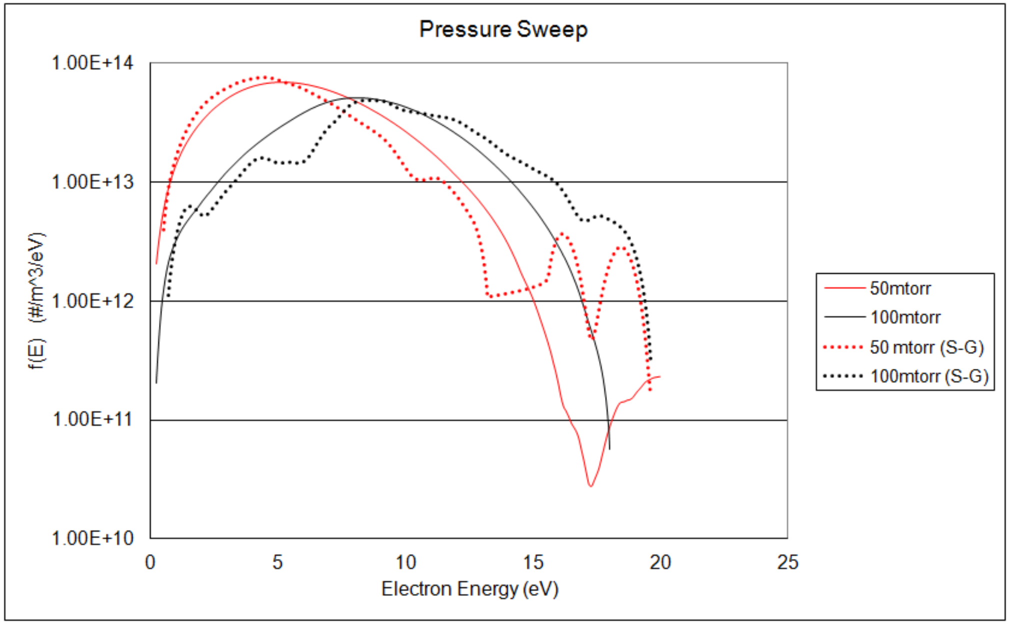


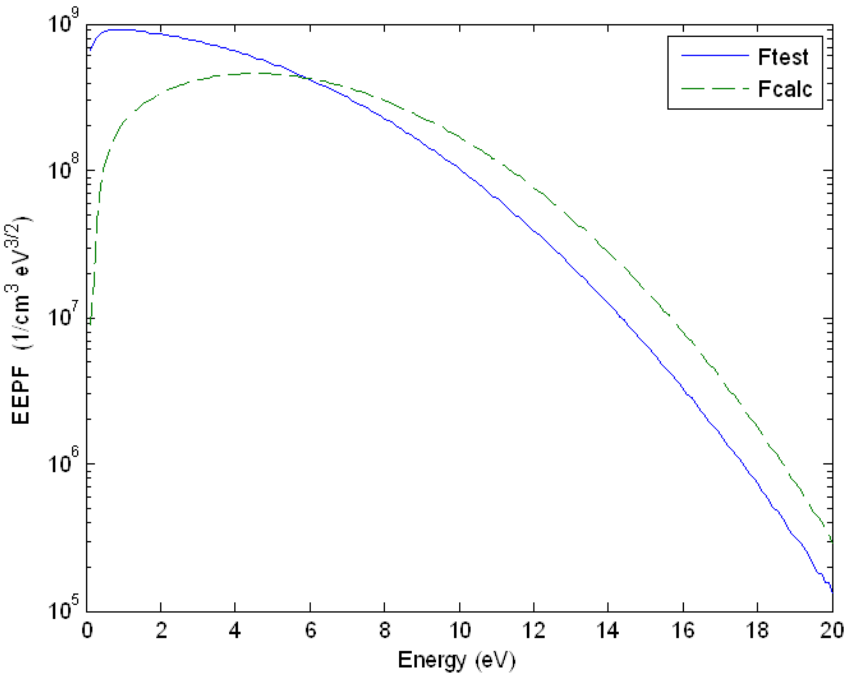
Maxwellian



Druyvesteyn

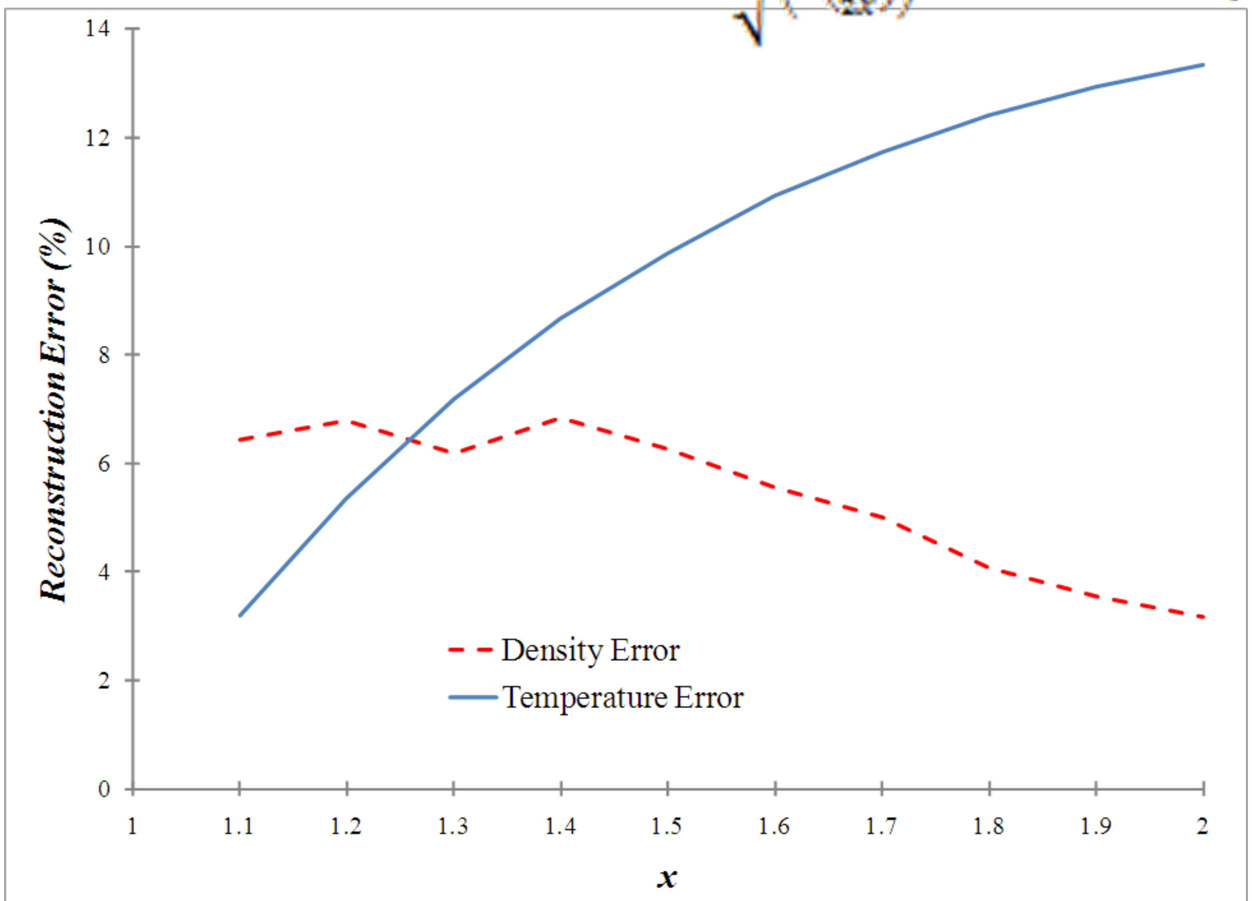
Distortion appears to get worse as distribution becomes less "Maxwellian"… supporting Emeleus work from the 1950's


Pressure Sweep
1.00E+14
1.00E+13
1.00E+12
1.00E+11
1.00E+10
f(E) (#/m^3/eV)
0
5
10
15
20
25
Electron Energy (eV)
50mtorr
100mtorr
50 mtorr (S-G)
100mtorr (S-G)


Ftest
Fcalc
EEPF (1/cm3 eV3/2)
Energy (eV)
0
2
4
6
8
10
12
14
16
18
20

Distortion vs. "Maxwellian-ness"

$$f_x(E) = c_1\sqrt{\frac{E}{T_x^{\,3}}}\,e^{-c_2\left(\frac{E}{T_x}\right)^x} \qquad c_1 = x\left(\frac{2}{3}\right)^{3/2}\sqrt{\frac{\left(\Gamma\left(\frac{5}{2x}\right)\right)^3}{\left(\Gamma\left(\frac{3}{2x}\right)\right)^5}} \qquad c_2 = \left(\frac{2\Gamma\left(\frac{5}{2x}\right)}{3\Gamma\left(\frac{3}{2x}\right)}\right)^x$$



Summary so far…

- Depending on your probe geometry, you will have a different integral function that defines the collection of electron current as a function of probe voltage
- The Druyvesteyn relation is derived for the spherical probe case, and assumes geometric invariance and applicability to other probe geometries
- Observations from other groups suggest that this may not be the case
- Computational efforts (shown here) suggest that this invariance assumption may have validity for Maxwellian type distributions, but may distort measured EEDF's for non-Maxwellian conditions

Using the cylindrical equation instead of the Druyvesteyn has it's challenges though…

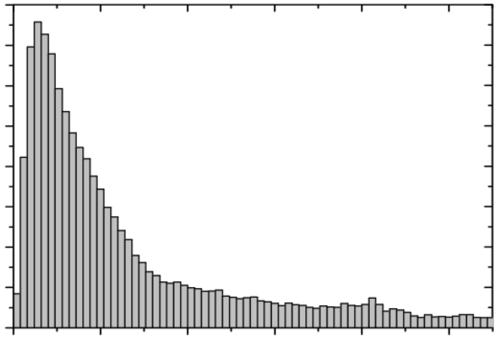
- There is no simple differential form of the equation that allows for EEDF analysis via simple data differentiation (of course, with noise amplification, I say simple and then laugh a little)
- Techniques for addressing experimental noise (data smoothing, Boyd Twitty method, point by point differentiation) have limited if any application for solving the integral form of the problem

One way to address this is to solve the integral problem instead of the differential problem

Calculating Electron Current vs. Probe Voltage for an arbitrary distribution function

Hystogram representation of the EEDF allows for the evaluation of non-Maxwellian EEDF's

$$f(\mathrm{E})=\sum_{i=1}^{N}\left\{\begin{array}{ll} 0 & \text{if} \quad \mathrm{E}<\mathrm{E}_{\mathrm{Li}} \\ \dfrac{n_{ei}}{\mathrm{E}_{\mathrm{Hi}}-\mathrm{E}_{\mathrm{Li}}} & \text{if} \quad \mathrm{E}_{\mathrm{Li}} \leq \mathrm{E} \leq \mathrm{E}_{\mathrm{Hi}} \\ 0 & \text{if} \quad \mathrm{E}>\mathrm{E}_{\mathrm{Hi}} \end{array}\right\}$$



Plug into the appropriate integral (spherical or cylindrical) to get electron current

$$I_e = -\frac{A_p e}{4}\sqrt{\frac{2}{m_e}} \int_{-eV_{probe}}^{\infty} \frac{f(E)\left(E+eV_{probe}\right)}{\sqrt{E}}\, dE$$

Or

$$I_e\left(V_p\right) = \frac{-A_p e}{4\pi} 2\pi \int_{E=V_p}^{\infty} f(E)\sqrt{\frac{2E}{m}} \int_{b=0}^{r_p\sqrt{1-\frac{V_p}{E}}} b\, db\, dE$$

Yes… they are ugly… the good news is that you only have to solve it one time!

$$I_e(E) = \sum_{i=1}^{N} \begin{cases} \dfrac{n_{ei} A_p e}{2(E_{Hi} - E_{Li})} \sqrt{\dfrac{2}{m_e}} \left[\left(\sqrt{E_{Li}} - \sqrt{E_{Hi}}\right) E + \dfrac{\sqrt{E_{Hi}^3} - \sqrt{E_{Li}^3}}{3} \right] & \text{if } E < E_{Li} \\ \dfrac{n_{ei} A_p e}{6(E_{Hi} - E_{Li})} \sqrt{\dfrac{2}{m_e}} \left[2\sqrt{E^3} - 3\sqrt{E_{Hi}} E + \sqrt{E_{Hi}^3} \right] & \text{if } E_{Li} \le E \le E_{Hi} \\ 0 & \text{if } E > E_{Hi} \end{cases}$$

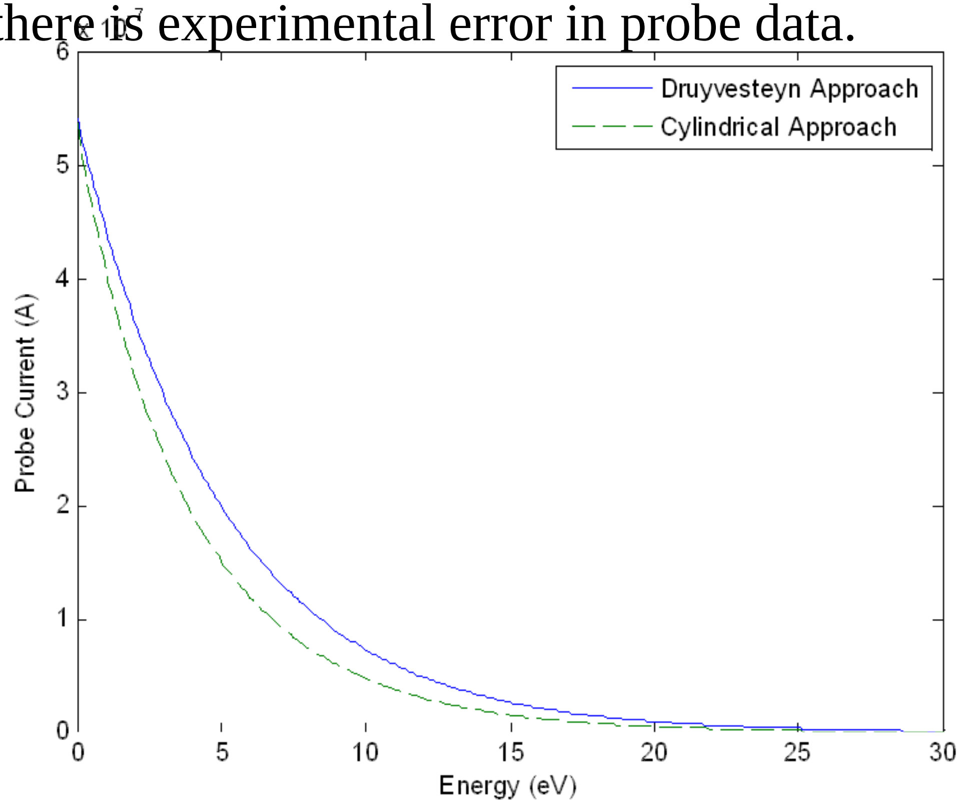
$$I_e = \begin{cases} \dfrac{2A_p e n_e}{3\pi(E_L + E_H)} \sqrt{\dfrac{2}{m}} (E_L - V_p)^{3/2} & E < E_L \\ \dfrac{2A_p e n_e}{3\pi(E_L^2 - E_H^2)} \sqrt{\dfrac{2}{m}} \left[\dfrac{1}{5}(3E_H + 2V_p)(E_H - V_p)^{3/2} - E_H(E_H - V_p)^{3/2} - \dfrac{1}{5}(3E_L + 2V_p)(E_L - V_p)^{3/2} + E_H(E_L - V_p)^{3/2} \right] & E_L < E < E_H \\ 0 & E_H < E \end{cases}$$

Reduces to a system of linear equations

Either of these solutions (spherical or cylindrical) is a linear system of equations that relates electron current I_e to EEDF f_e by a kernel K

$$I_e = K f_e$$

Unfortunately, this is a very ill-conditioned problem that is not very forgiving when there is experimental error in probe data.



This technique enables use of algorithms that are very good at handling noise amplifcation and subsequent distortion and error, ElSaghir, Shannon (2011). *IEEE Trans. Plasma Sci.* ***39****(1)* pp. 596-602

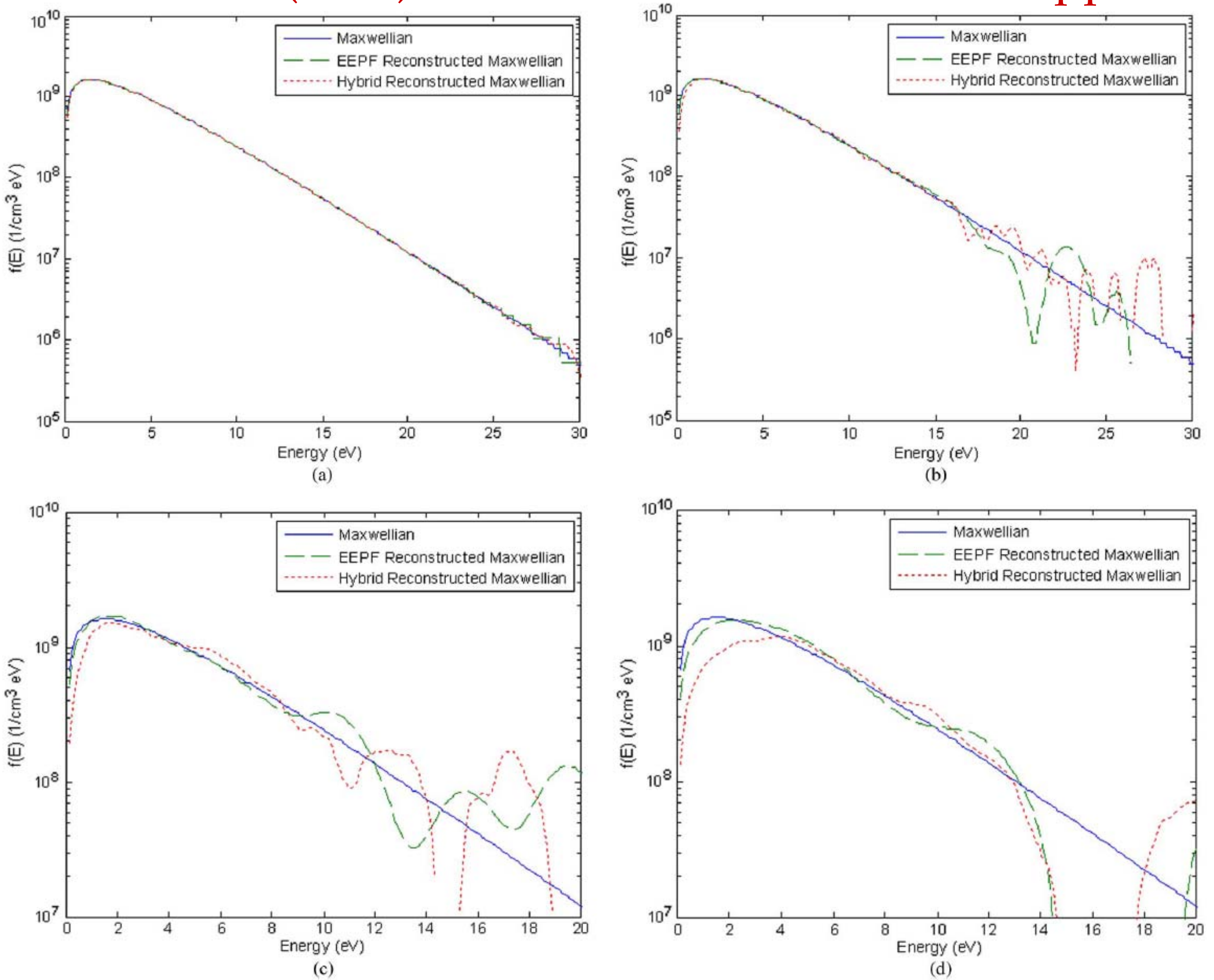


Fig. 8. Comparison between the hybrid method and the EEPF Tikhonov method for different SNR values. (a) SNR = 100, (b) SNR = 50, (c) SNR = 20, (d) SNR = 10.

Great! We'll just use this new integral and everything will be just fine!

Not so fast… remember noise amplification for the second derivative problem? It gets worse for this one…

Simple rule of thumb… the faster the singular values decay, the greater the influence of noise on your reconstruction

Things can get tricky at moderate noise levels

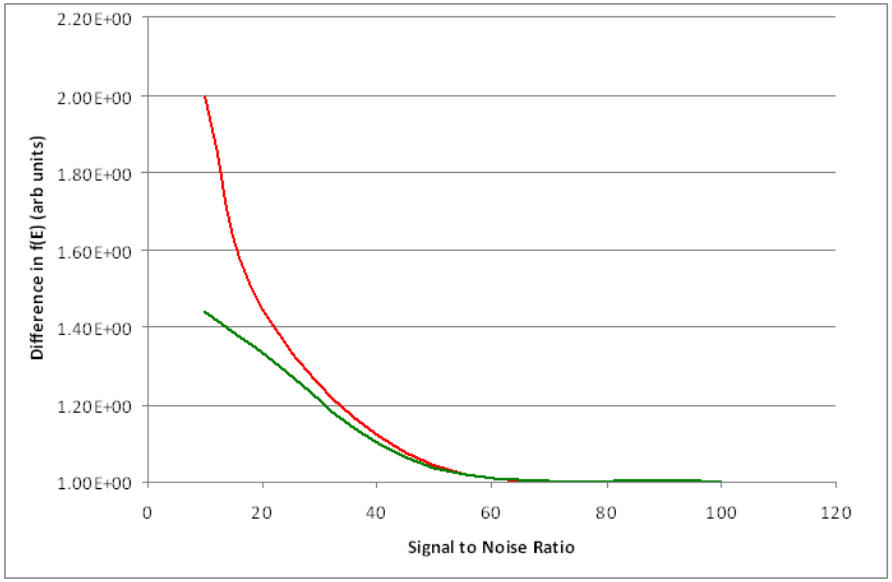
Take a Maxwellian

Calculate I_e from Druyvesteyn

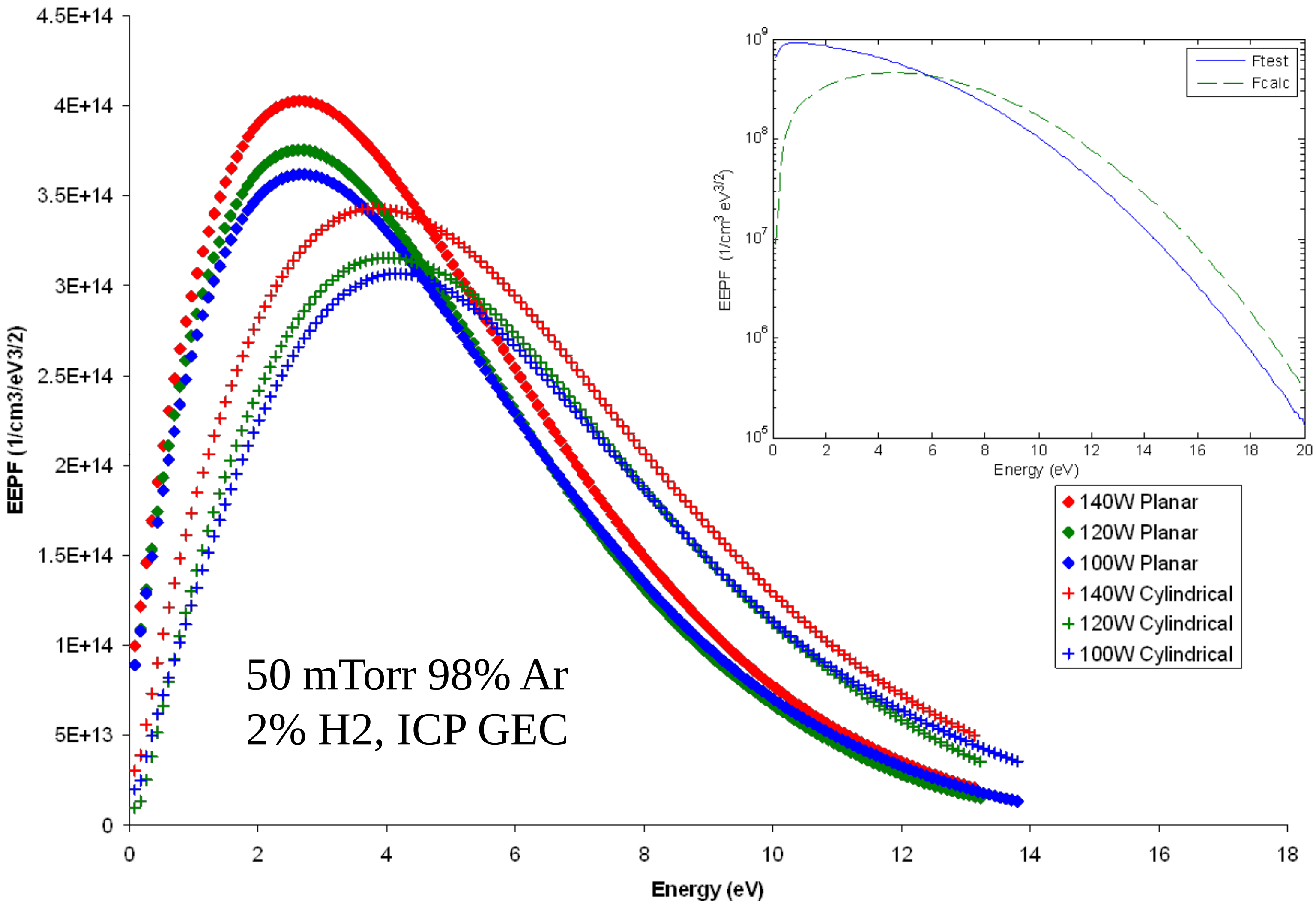
Add Noise

Solve for *f(E)*

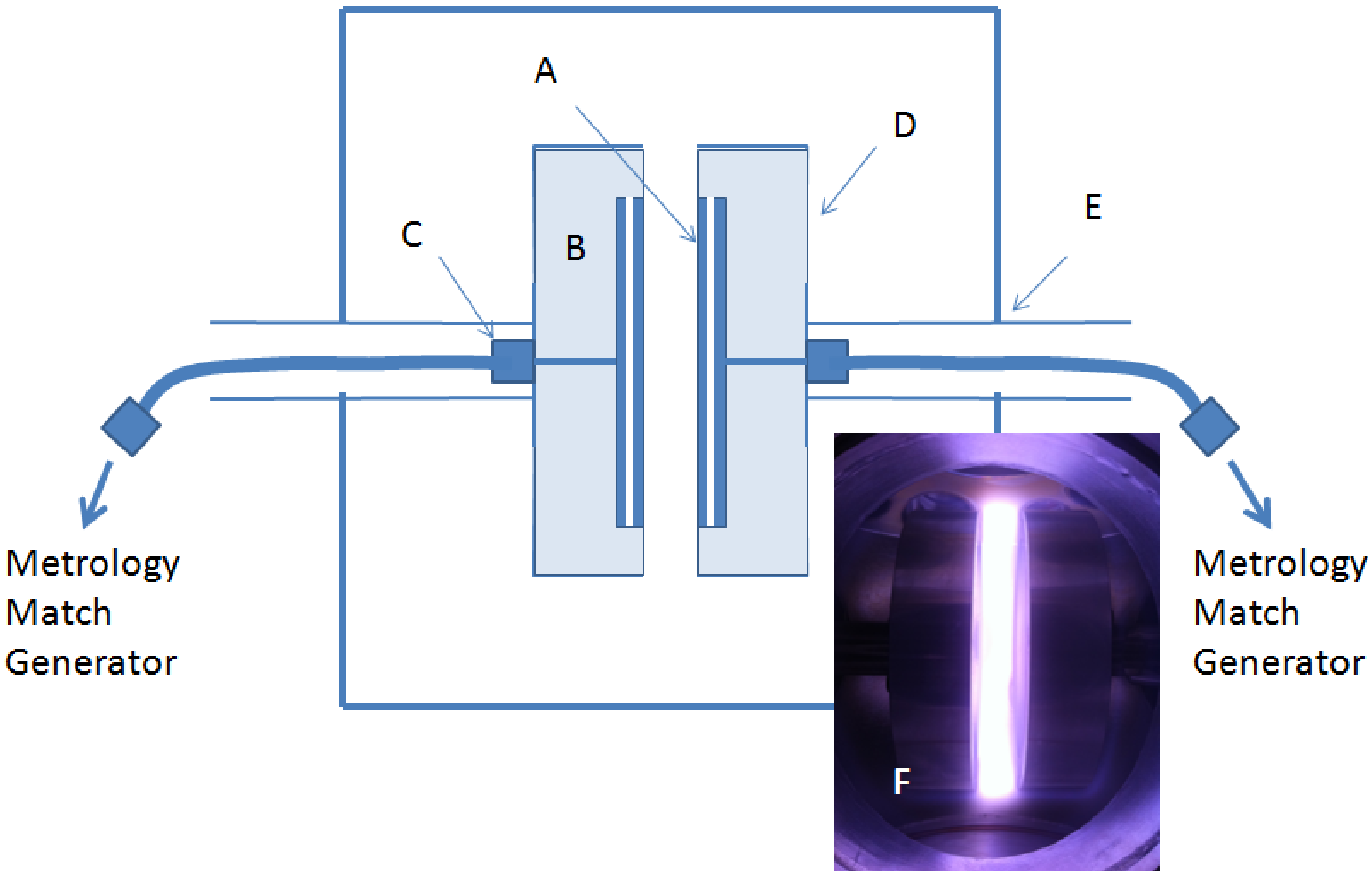
Compare



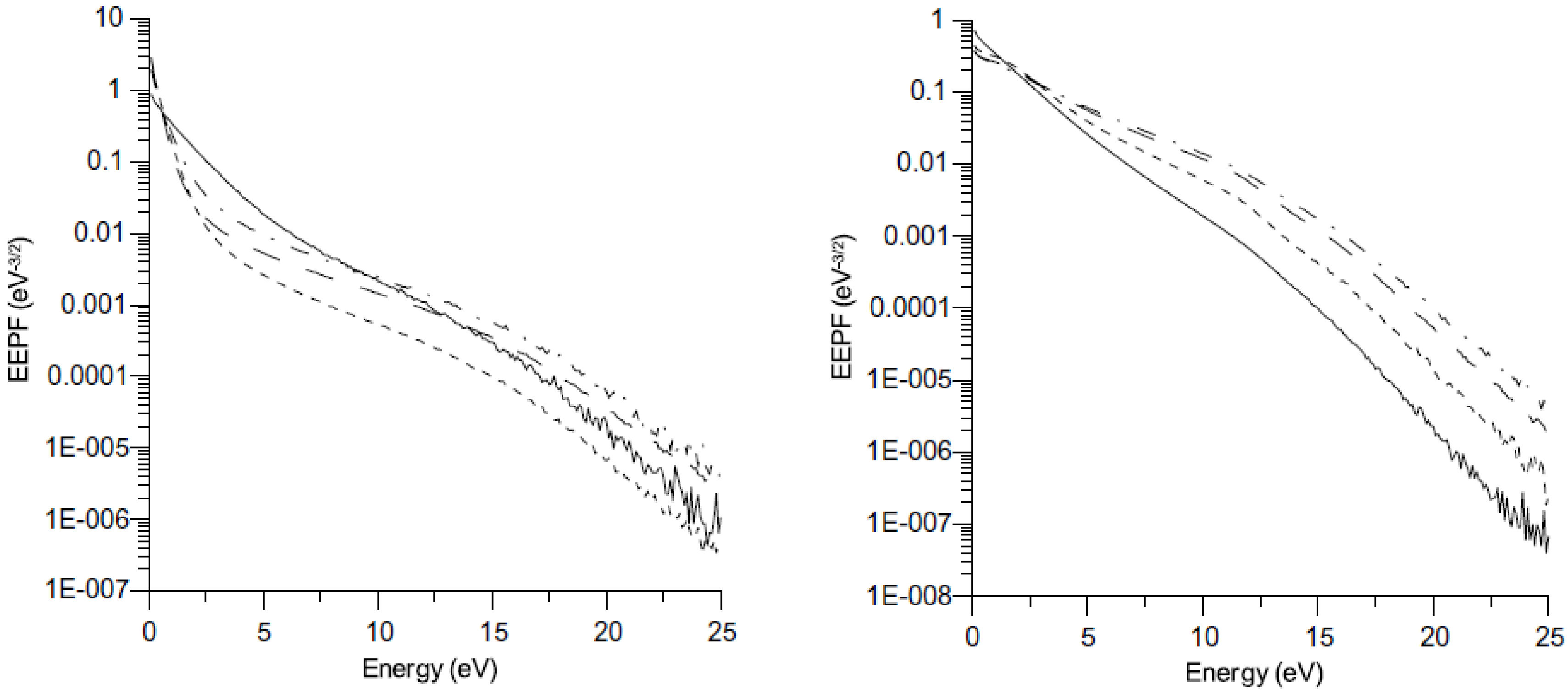
Preliminary measurements in Ar/H_2 plasmas



Experimental setup for more detailed study and comparison to computed EEDF's



PIC simulation of CCP systems show strong EEDF shift with respect to hydrogen content



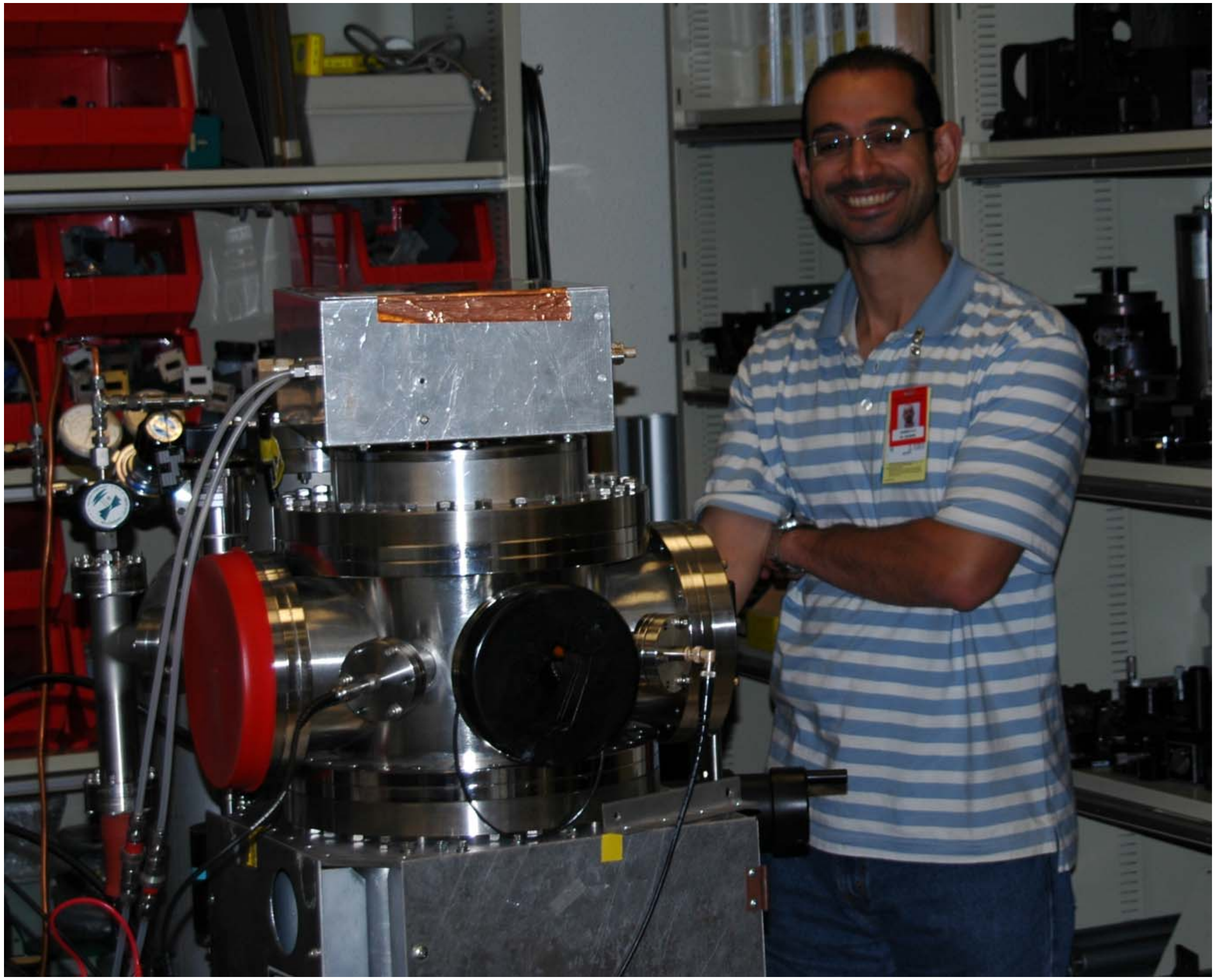
Neyts, Yan, Bogaerts, Gijbels. *Nucl. Instrum. and Methods in Phys. Res. B* **202** (2003) 300–304

Conclusions and Future Work

- Geometry dependence of the relationship between electron current and EEDF has been derived for cylindrical probe geometries
- Distortion of EEDF derivation from cylindrical probe current when not accounting for probe geometry has been demonstrated
- No free lunch – the integral problem is an even bigger pain to deal with, and can be worse than the systematic error generated by the integral formulation

- Currently comparing probe geometries in a system with non-Maxwellian functions to validate

References and Acknowledgements



References and Acknowledgements

- Boyd R L F and Twiddy N D 1959 Proc. R. Soc. A **53** 250
- Druyvesteyn; *Physica* **10** 61 (1930)
- El Saghir, Kennedy, Shannon (2010). *IEEE Trans. Plasma Sci.* ***38****(2)* pp. 156-162
- El Saghir, Shannon (2011), *IEEE Trans. Plasma Sci.* ***39****(1)* pp. 596-602
- Emeleus; *Physical Letters* **71A**, Nos. 2,3 1979
- Hoskinson, Hershkowitz (2006). *Plasma Sources Sci. Technol*. **15** pp. 85–9
- Kagan and Perel; Usp. Fiz. Nauk Vol 81, pages 409-452 (Dec. 1963
- Knappmiller, Robertson, Sternovsky; Phys. Rev. E **73**, 066402 (2006)
- Mott-Smith and Langmuir (1926). *Physical Review* **28**(4), 727-763
- Neyts, Yan, Bogaerts, Gijbels. *Nucl. Instrum. and Methods in Phys. Res. B* **202** (2003) 300–304

Acknowledgements

A world of thanks to Ed Barnat, Mohamed Bourham, John Gilligan, Valery Godyak, Greg Hebner, Noah Hershkowitz, Paul Miller, Scott Robertson, and the many others who have provided constructive feedback and guidance on this over the past year (as well as a great experiment in Ed’s lab)

A universe of thanks to Ahmed El Saghir for doing all the work

And a special thanks to the North Carolina General Assembly, Applied Materials, and Sandia National Laboratory for support on this project